AIEEE - 2011

Section : Mathematics

1. Consider 5 independent Bernoulli's trails each with probability of sucess p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(1) $\left(\frac{11}{12}, 1\right]$ **(2)** $\left(\frac{1}{2}, \frac{3}{4}\right]$

(3) $\left(\frac{3}{4}, \frac{11}{12}\right]$ **(4)** $\left(0, \frac{1}{2}\right]$

Ans: [4]

2. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is

(1) 132 **(2)** 144

(3) – 132 **(4)** – 144

Ans: [4]

3. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(1) equals $\frac{1}{\sqrt{2}}$ **(2)** does not exist

(3) equals $\sqrt{2}$ **(4)** equals $-\sqrt{2}$

Ans: [2]

4. Let R be the set of real numbers

Statement -1

$A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R.

Statement -2

$B = \{(x, y) \in R \times R : x - \alpha y$ for some rational number α } is an equivalence relation on R.

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: [4]

5. Let α, β be real and z be complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that

(1) $\beta \in (1, \infty)$ **(2)** $\beta \in (0, 1)$

(3) $\beta \in (-1, 0)$ **(4)** $|\beta| = 1$

Ans: [1]

6. $\frac{d^2x}{dy^2}$ equal

(1) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$ **(2)** $\left(\frac{d^2y}{dx^2}\right)^{-1}$

(3) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ **(4)** $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

Ans: [1]

7. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is

(1) zero **(2)** 3

(3) 2 **(4)** 1

Ans: [3]

8. **Statement -1**

The point A (1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Statement -2

The line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A (1, 0, 7) and B (1, 6, 3)

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: [2]

9. Consider the following statements

P : Suman is brilliant

. Q : Suman is rich

R : Suman is honest

The negation of the statment "Suman is brilliant and dishonest if and only if suman is rich" can be expressed as

(1) $\sim(P \wedge \sim R) \leftrightarrow Q$ **(2)** $\sim P \wedge (Q \leftrightarrow \sim R)$

(3) $\sim(Q \leftrightarrow (P \wedge \sim R))$ **(4)** $\sim Q \leftrightarrow \sim P \wedge R$

Ans: [3]

10. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectivley. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement -1

The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

Statement -2

In any triangle, bisector an angle divides the trianle into two similar triangles.

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: [4]

11. A man saves Rs 200 in each of the first three months of his services. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after

(1) 21 months **(2)** 18 months

(3) 19 months **(4)** 20 months

Ans: [1]

12. Equations of the ellipse whose axes are the axes of coordinates and which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{2/5}$ is

(1) $5x^2 + 3y^2 - 32 = 0$ **(2)** $3x^2 + 5y^2 - 32 = 0$

(3) $5x^2 + 3y^2 - 48 = 0$ **(4)** $3x^2 + 5y^2 - 15 = 0$

Ans: [2]

13. If $A = \sin^2 x + \cos^4 x$ then for all real x.

(1) $\frac{3}{4} \le A \le \frac{13}{16}$ **(2)** $\frac{3}{4} \le A \le 1$

(3) $\frac{13}{16} \le A \le 1$ **(4)** $1 \le A \le 2$

Ans: [2]

14. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}\,dx$ is

(1) $\log 2$ **(2)** $\pi \log 2$

(3) $\frac{\pi}{8}\log 2$ **(4)** $\frac{\pi}{2}\log 2$

Ans: [2]

15. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$

(1) $5/3$ **(2)** $2/3$

(3) $3/2$ **(4)** $2/5$

Ans: [2]

16. For $x \in \left(0, \frac{5\pi}{2}\right)$, define

$$f(x) = \int_0^x \sqrt{t}\ \sin t\, dt$$

Then f has

(1) local maximum at π and local minimum at 2π

(2) lcoal maximum at π and 2π

(3) lcoal minimum at π and 2π

(4) local minimum at π and local maximum at 2π

Ans: [1]

17. The domain of the function

(1) $f(x) = \frac{1}{\sqrt{|x| - x}}$ is **(2)** $(-\infty, \infty) - \{0\}$

(3) $(0, \infty)$ **(4)** $(-\infty, 0)$

Ans: [4]

18. If the mean deviation about the median of the numbers a, 2a, . . ., 50 a is 50, then $|a|$ equals

(1) 5 **(2)** 2
(3) 3 **(4)** 4

Ans: [4]

19. If $a = \frac{1}{\sqrt{10}}(3i + k)$ and $b = \frac{1}{7}(2i + 3j - 6k)$, then the value of $(2a - b) \cdot [(a \times b)(a + 2b)]$ is

(1) 3 **(2)** -5
(3) -3 **(4)** 5

Ans: [2]

20. The value of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \frac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$$

is continuous for all x in R, are,

(1) $p = \frac{1}{2}, q = \frac{3}{2}$ **(2)** $p = \frac{1}{2}, q = -\frac{3}{2}$

(3) $p = \frac{5}{2}, q = \frac{3}{2}$ **(4)** $p = -\frac{3}{2}, q = \frac{1}{2}$

Ans: [4]

21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if

(1) $|a| = 2c$ **(2)** $2|a| = c$

(3) $|a| = c$ **(4)** $a = 2c$

Ans: [3]

22. Let be the purchase value of an equaipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential euqation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is

(1) e^{-kT} **(2)** $T^2 - \frac{1}{k}$

(3) $I - \frac{kT^2}{2}$ **(4)** $I - \frac{k(T-t)^2}{2}$

Ans: [3]

23. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(1) $P(C|D) = \frac{P(D)}{P(C)}$ **(2)** $P(C|D) = P(C)$

(3) $P(C|D) \geq P(C)$ **(4)** $P(C|D) < P(C)$

Ans: [3]

24. Let A and B be two symmetric matrices of order 3

Statement -1

A (BA) and (AB) A are symmetric matrices

Statement -2

AB is symmetric matrix if matrix multiplication of A with B commutative.

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: [3]

25. If $\omega(\neq 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals

(1) $P(C|D) = \frac{P(D)}{P(C)}$ **(2)** $P(C|D) = P(C)$

(3) $P(C|D) \geq P(C)$ **(4)** $P(C|D) < P(C)$

Ans: [3]

26 **Statement -1**

The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^{9}C_3$.

Statement -2

The number of ways of choosing any 3 places from 9 different places is ${}^{9}C_3$

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: [3]

27 The shortest distance between line $y-x=1$ and curve $x=y^2$ is

(1) $\frac{4}{\sqrt{3}}$ **(2)** $\frac{\sqrt{3}}{4}$

(3) $\frac{3\sqrt{2}}{8}$ **(4)** $\frac{8}{3\sqrt{2}}$

Ans: [3]

28. The area of the region enclosed by trhe curves $y=x, x=e, y=(1/x)$ and the positive x-axis is

(1) 5/2 square units

(2) 1/2 square units

(3) 1 square units

(4) 3/21 square units

Ans: [4]

29. If $\frac{dy}{dx}=y+3>0$ and $y(0)=2$, the $y(\ln 2)$ is equal to:

(1) –2 **(2)** 7

(3) 5 **(4)** 13

Ans: [2]

30. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d}=0$. Then the vector $\vec{d}$ is equal to:

(1) $\vec{c}-\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ **(2)** $\vec{b}-\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

(3) $\vec{c}+\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ **(4)** $\vec{b}+\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

Ans: [1]