Solutions

1. $x = 16^3 + 17^3 + 18^3 + 19^3$

$= (16^3 + 19^3) + (17^3 + 18^3)$

$= (16 + 19)(16^2 + 16 \times 19 + 19^2) + (17 + 18)(17^2 + 17 \times 18 + 18^2)$

= 35 × (an odd number) + 35 × (another odd number) = 35 × (an even number)

= 35 × (2k) … (k is a positive integer)

$\therefore x = 70k$

$\therefore x$ is divisible by 70.

Remainder when x is divided by 70 = 0

Hence, option 1.

2. The change in the amount of chemical in each tank after every minute is as follows:

A:$-20-10+90=60$

B:$-100+110+20=30$

C:$-50-90+100=-40$

D:$-110+10+50=-50$

Since tank D loses the maximum amount of chemical in a minute, it will be emptied first.

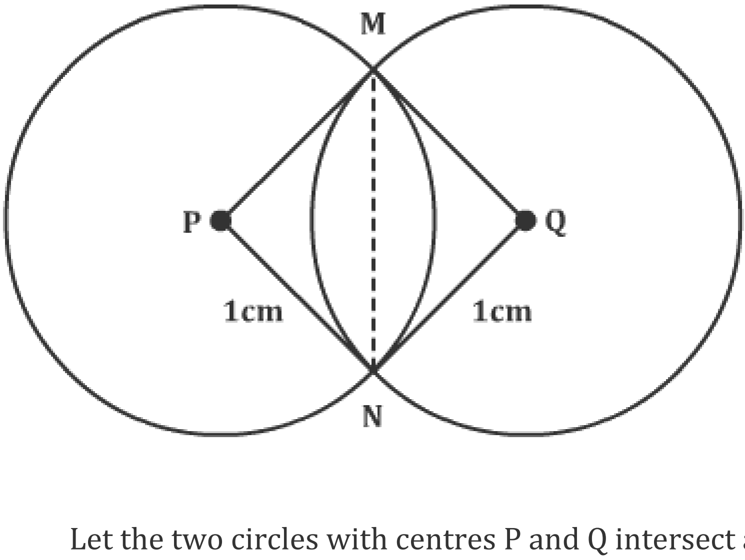
Let n minutes be the time taken by tank D to get empty.

$\therefore 1000-50n=0$

$\therefore n=20$minutes

Hence, option 3.

3.



Let the two circles with centres P and Q intersect at M and N.

Quadrilateral PQMN is a square.

m∠ MPN = m∠ MQN = 90°

The area common to both the circles = 2(Area of sector P-MN – Area of Δ PMN)

$= 2[(90/360 \times \pi \times 1^2) - (1/2 \times 1^2)]$
$= \pi/2 - 1$
Hence, option 2.

4. Let r be the radius of the circular tracks.
Length and breadth of the rectangular track are $4r$ and $2r$ respectively.
Length (perimeter) of the rectangular track = $12r$
Length of the two circular tracks (figure of eight) = 4r
If A and B have to reach their starting points at the same time,
$$\frac{12r}{a} = \frac{4\pi r}{b}$$
(where a and b are the speeds of A and B respectively)
$$\therefore \frac{b}{a} = \frac{4\pi}{12}$$
$\therefore (b - a) \times 100/a = 0.047 \times 100$
= 4.7%
Hence, option 4.

5. Let there be g girls and b boys.
Number of games between two girls = ${}^{g}C_2$
Number of games between two boys = ${}^{b}C_2$
$\therefore g(g - 1)/2 = 45$
$\therefore g^2 - g - 90 = 0$
$\therefore (g - 10)(g + 9) = 0$
$\therefore g = 10$

Also,
$b(b - 1)/2 = 190$
$\therefore b^2 - b - 380 = 0$
$\therefore (b + 19)(b - 20) = 0$
$\therefore b = 20$
$\therefore$ Total number of games = ${}^{(g+b)}C_2 = {}^{30}C_2 = 435$
$\therefore$ Number of games in which one player is a boy and the other is a girl = 435 – 45 – 190 = 200
Hence, option 1.

6. Ram and Shyam run a race between points A and B, 5 km apart. Ram starts at 9 a.m. from A at a speed of 5 km/hr, reaches B, and returns to A at the same speed. Shyam starts at 9:45 a.m. from A at a speed of 10 km/hr, reaches B and comes back to A at the same speed.
Ram starts at 9:00 am and Shyam starts at 9:45 am from A.
Ram reaches B at 10:00 am (as his speed is 5km/hr and the distance between A and B is 5km)
When Ram reaches B, Shyam is 15/60 × 10 = 2.5 km away from A.
Ram meets Shyam (2.5 × 60)/(10 + 5) minutes after 10:00 a.m. i.e., at 10:10 a.m.
Shyam reaches B at 10:15 a.m.
At 10:15 a.m., Ram is (15/60) × 5 = 1.25 km away from B.
Shyam overtakes Ram in 1.25/(10 – 5) = 0.25 hrs = 15 minutes after 10:15 am i.e. at 10:30 a.m.

Hence, option 2.

7. Shyam overtakes Ram at 10:30 a.m.

Hence, option 2.

8.

$$R = \frac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$$

$\because a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + b^{n-1})$

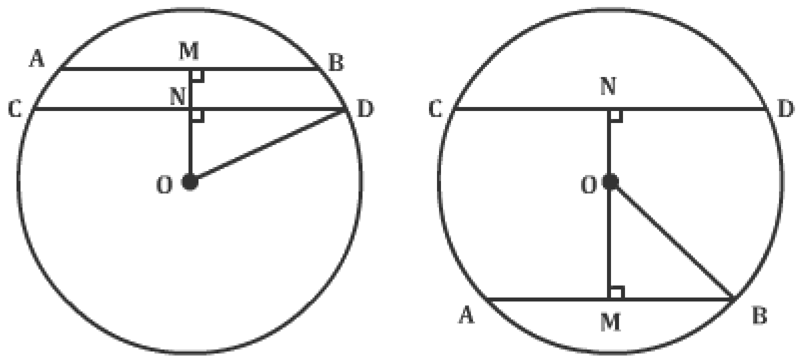
$$\therefore R = \frac{(30 - 29)[30^{64} + (30^{63} \times 29) + \cdots + 29^{64}]}{30^{64} + 29^{64}}$$

$\because 30^{64} + 30^{63} \times 29 + \cdots + 29^{64} > 30^{64} + 29^{64}$

$\therefore R > 1$

Hence, option 4.

9.



The two chords AB and CD can be on the same side or the opposite sides of the centre O.
Let M and N be the midpoints of AB and CD.
∴ MN is the distance between the two chords.
MB = 12 cm and ND = 16 cm
OM and ON are perpendicular to AB and CD respectively.
∴ $ON^2 = 20^2 - 16^2$ (By Pythagoras theorem)
∴ ON = 12 cm
Similarly, OM = 16 cm

Case 1: AB and BC are on the same side of the centre.

MN = OM – ON = 4 cm

Case 2: MN = OM + ON = 28 cm

Hence, option 4.

10. We have, $x^2 = y^2$ and $(x - k)^2 + y^2 = 1$

Solving the two equations simultaneously, we get,

$(x - k)^2 + x^2 = 1$

$\therefore x^2 - 2kx + k^2 + x^2 = 1$

$\therefore 2x^2 - 2kx + (k^2 - 1) = 0$

If this equation has a unique solution for x, then discriminant = 0

$\therefore 4k^2 - 8(k^2 - 1) = 0$

$\therefore 8 - 4k^2 = 0$

$\therefore k^2 = 2$

$\therefore k = \pm\sqrt{2}$

Since k is positive the other solution is ruled out

$\therefore k = \sqrt{2}$

Hence, option 3.

11. $p = (1 \times 1!) + (2 \times 2!) + (3 \times 3!) + (4 \times 4!) + \ldots + (10 \times 10!)$

Now, $n \times n! = [(n + 1) - 1] \times n! = (n + 1)! - n!$

$\therefore p = 2! - 1! + 3! - 2! + 4! - 3! + 5! - 4! + \ldots + 11! - 10!$

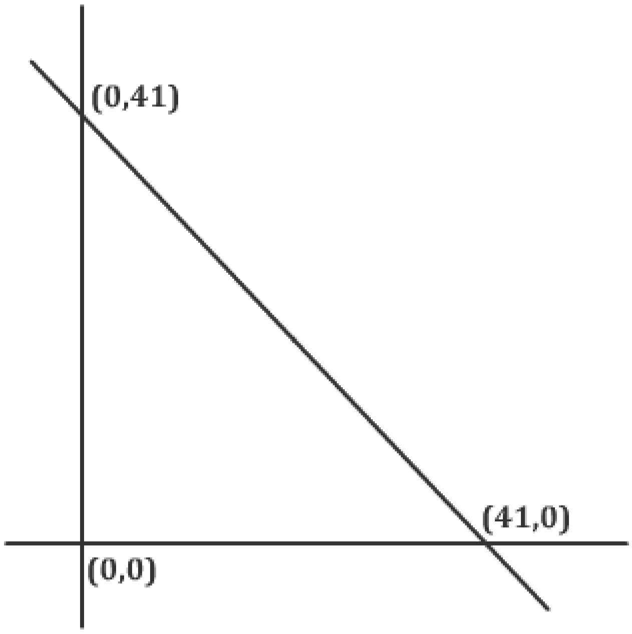
$\therefore p = 11! - 1! = 11! - 1$

$\therefore p + 2 = 11! + 1$

$\therefore p + 2$ when divided by 11! leaves a remainder of 1.

Hence, option 4.

12.



The points satisfying the equations $x + y < 41$, $y > 0$, $x > 0$ lie inside the triangle.
Integer solutions of $x + y < 41$ can be found as follows.
If $x + y = 40$
(x, y) (1, 39), (2, 38), …, (39, 1) ... (39 solutions)
If $x + y = 39$
(1, 38), (2, 37), …, (38, 1) ... (38 solutions)
If $x + y = 38$, we get 37 solutions and so on till $x + y = 2$ ... (1 solution)
Thus there are $39 \times 40/2 = 780$ integer solutions to $x + y < 41$
The number of points with integer coordinates lying inside the circle = 780
Hence, option 1.

13. Let A = $100x + 10y + z$ ($x \neq 0$, x, y, z are single digit numbers)
$\therefore$ B = $100z + 10y + x$
$\therefore$ B – A = $99(z - x)$
As (B – A) is divisible by 7 and 99 is not, $(z - x)$ is divisible by 7
$\therefore$ z and x can have values (8, 1) or (9, 2)
y can have any value from 0 to 9.
A = $1y8$ or $2y9$
$\therefore$ Lowest possible value of A is 108 and the highest possible value of A is 299.
Hence, option 2.

14. $a_1 = 1$
$a_{n+1} = 4n + 3a_n - 2$
$a_2 = 4 - 2 + 3(1) = 5 = 3^2 - 1$
$a_3 = 4(2) + 3(5) - 2 = 21 = 3^3 - 6$
$a_4 = 4(3) + 3(21) - 2 = 73 = 3^4 - 8$
$\therefore a_n = 3^n - 2(n)$

∴ $a_{100} = 3^{100} - 200$

Hence, option 3.

15. Let O and E represent odd and even digits respectively.
∴ S can have digits of the form
O _ O _ E or O _ E _ O or E _ O _ O

Case 1: O _ O _ E
The first digit can be chosen in 3 ways out of 1, 3 and 5
The third can be chosen in 2 ways.
The fifth digit can be chosen in 2 ways after which the second and fourth digits can be chosen in 2 ways.
∴ There are 3 × 2 × 2 × 3 = 24 ways in which this number can be written. 12 out of these ways will have 2 in the rightmost position and 12 will have 4 in the rightmost position.
∴ The sum of the rightmost digits in Case 1 = (12 × 2) + (12 × 4) = 72

Case 2: O _ E _ O
This number can again be written in 24 ways.
8 out of these ways will have 1 in the rightmost position, 8 will have 3 in the rightmost position and 8 will have 5 in the rightmost position.
Thus the sum of the rightmost digits in Case 2 = (8 × 1) + (8 × 3) + (8 × 5) = 72

Case 3: E _ O _ O
This number can also be written in 24 ways.
As in Case 2, 8 out of these ways will have 1 in the rightmost position, 8 will have 3 in the rightmost position and 8 will have 5 in the rightmost position.
∴ The sum of the rightmost digits in Case 3 = (8 × 1) + (8 × 3) + (8 × 5) = 72
∴ The sum of the digits in the rightmost position of the numbers in S = 72 + 72 + 72 = 216
Hence, option 2.

16. $30^{2720} = 3^{2720} \times 10^{2720}$

The rightmost non zero digit of 30^{2720} will be the digit in the unit's place of 3^{2720}.

3's power cycle is 3, 9, 7, 1 and cyclicity is 4.

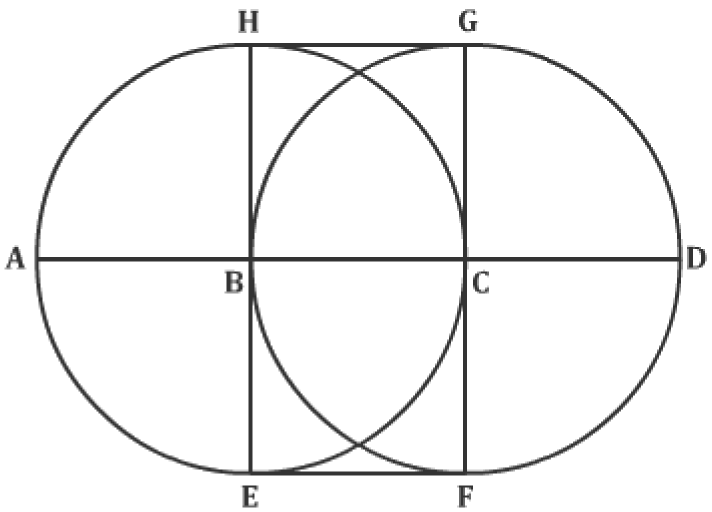
2720 = 680 × 4

∴ The digit in the unit's place of 3^{2720} is 1.

∴ The rightmost non-zero digit of 30^{2720} is 1.

Hence, option 1.

17.



The ant will not go into the circles with centers B and C and radius = 1 m
The minimum distance that the ant has to traverse = the distance of the path A-H-G-D
HG = 1m

$\text{AH} = \text{GD} = \frac{1}{4} \times \text{Circumference of Circle} = \frac{\pi}{2}$

AH + GD = π m

∴ The ant must traverse 1 + π m
Hence, option 2.

18.

$\log_x \left(\frac{x}{y}\right) + \log_y \left(\frac{y}{x}\right) = \frac{\log x - log\, y}{\log x} + \frac{log\, y - log\, x}{\log y}$

$=1 - \log_x y + 1 - \log_y x$

$= 2 - \log_x y - \log_y x$

$= 2 - (\log_x y + \log_y x)$

As $x \geq y$ and $y > 1$,

$log_y x \ \geq 1 \ and \ log_x y \leq 1$

$log_y x \ + \ log_x y > 1$

$\therefore \ 2 - (\log_x y + \log_y x) < 1$

$\therefore \log_x \left(\frac{x}{y}\right) + \log_y \left(\frac{y}{x}\right) \neq 1$

Hence, option 4.

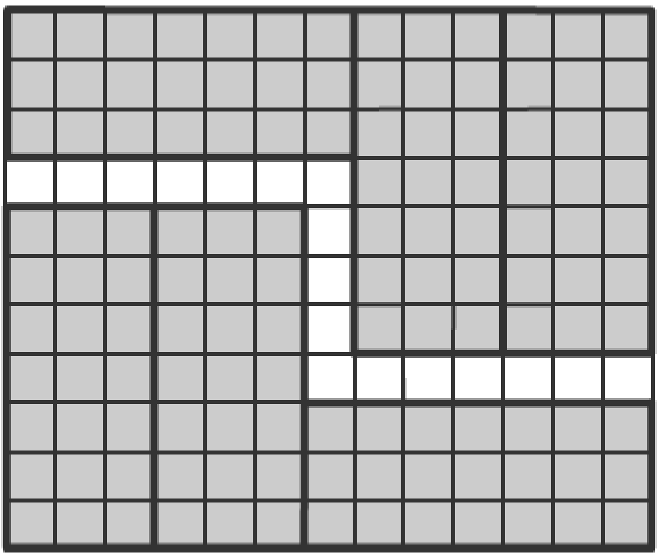
19. n can be a 2 digit or a 3 digit number.

Case (I)
Let n be a 2 digit number.
Let $n = 10x + y$, where x and y are non-negative integers,
$P_n = xy$ and $S_n = x + y$
Now, $P_n + S_n = n$
$\therefore xy + x + y = 10x + y$
$\therefore xy = 9xy = 9$
There are 9 two digit numbers (19, 29, 29, … ,99) for which $y = 9$

Case (II)
Let n be a 3 digit number.
Let $n = 100x + 10y + z$, where x, y and z are non-negative integers,
$P_n = xyz$ and $S_n = x + y + z$
Now, $P_n + S_n = n$
$xyz + x + y + z = 100x + 10y + z$
$\therefore xyz = 99x + 9y$
$\therefore z = 99/y + 9/x$

From the above expression, $0 < x, y < 9$
But, we cannot find any value of x and y, for which z is a single digit number.
$\therefore$ There are no 3 digit numbers which satisfy $P_n + S_n = n$
Hence, option 4.

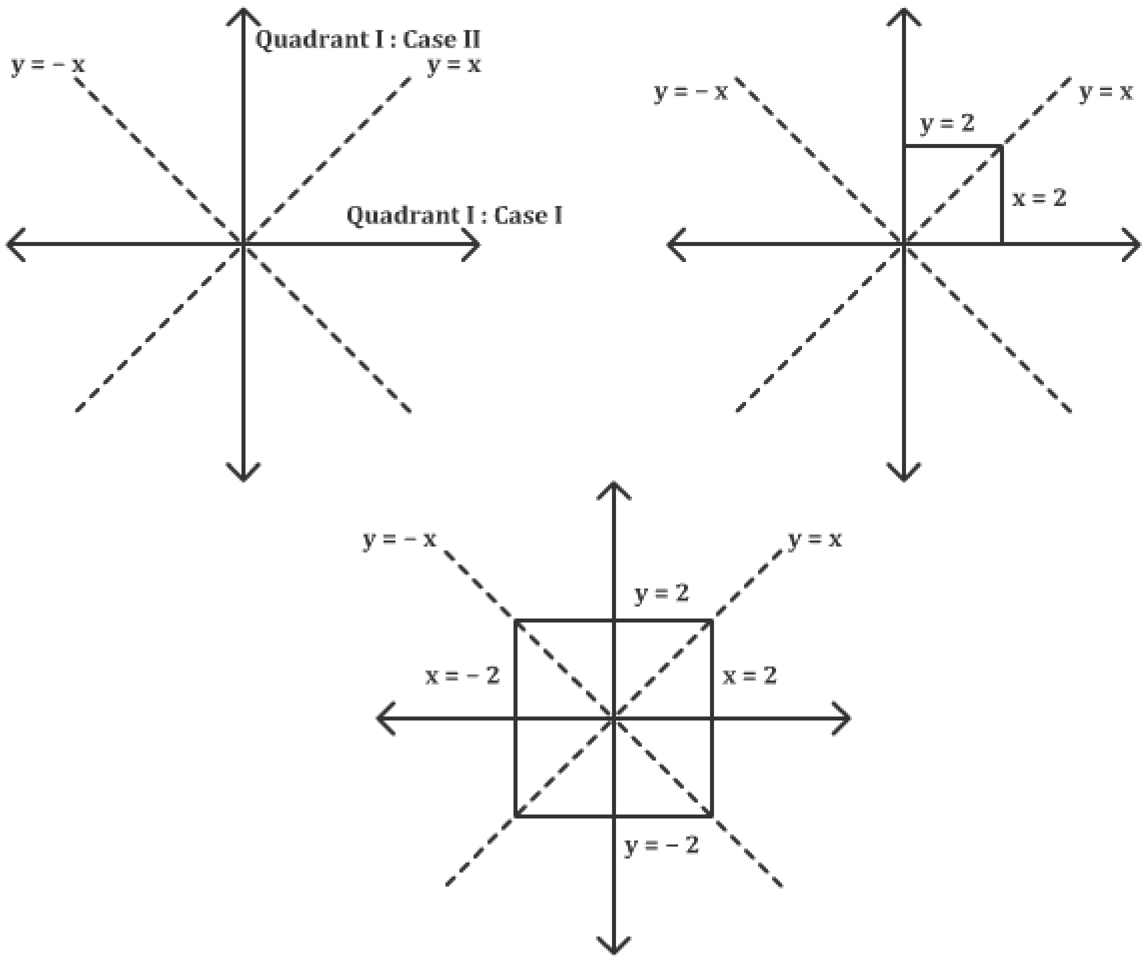
20.



This problem can be solved by trying different ways of placing the tiles on the floor. The maximum number of tiles that can be accommodated is 6 as shown in the figure.

Hence, option 3.

21.



$|x + y| + |x - y| = 4$ … (i)

Consider the case when x and y are both positive. This is the area of quadrant I
In this case two cases are possible.

Case I: $x > y$
In this case expression (i) becomes
$x + y + x - y = 4$
$\therefore 2x = 4$
$\therefore x = 2$

Case II: $x < y$
In this case expression (i) becomes
$x + y + y - x = 4$

∴ $2y = 4$
∴ $y = 2$
∴ The area for the first quadrant is as shown in the figure.
Extending the same logic to other quadrants we get the area as shown in the diagram.
∴ Its area = 4 × 4 = 16 sq. units

Hence, option 3.

22. AO = OD = 1.5 cm
AE + EB = 3cm and AE:EB = 1:2
∴ AE = 1 cm and EB = 2 cm
OE = AO – AE = 1.5 – 1 = 0.5 cm
Similarly, NL = 1 cm, M = 2 cm and OL = 0.5 cm
OEHL is a square as all its angles are right angles and OE = OL
∴ EH = HL = 0.5 cm
In Δ ODL, $\text{OD}\hat{}2 = \text{OL}\hat{}2 + \text{DL}\hat{}2$
$1.5^2 = 0.5^2 + (0.5 + \text{DH})^2$
$2.25 = 0.25 + 0.25 + \text{DH} + \text{DH}^2$

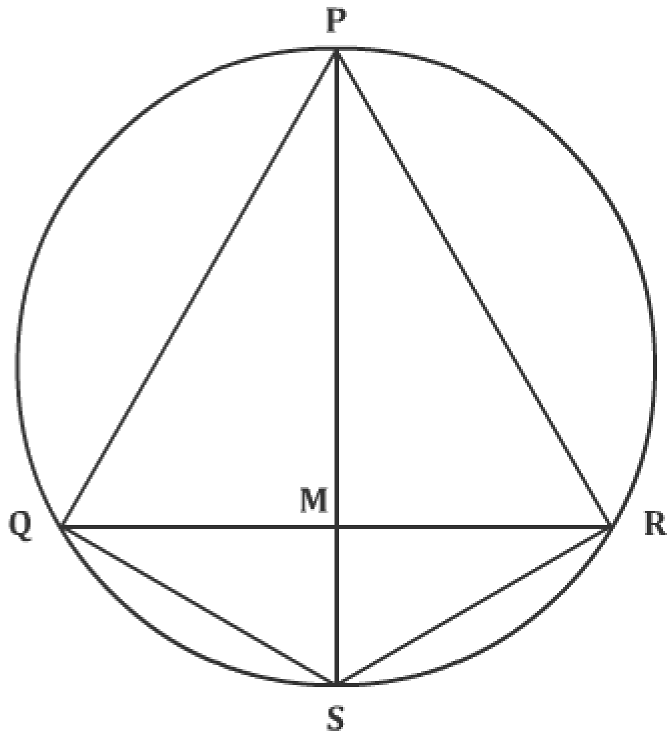
$\text{DH}^2 + \text{DH} - 1.75 = 0$
$\text{DH} = (-1 \pm \sqrt{}(1 - 4(-1.75)))/2$
$= (2\sqrt{2} - 1)/2 \qquad (\text{DH} > 0)$

Hence, option 2.

23. $m\angle$BCD = $m\angle$BAC and B is common to triangles ABC and CBD.
Δ ABC is similar to Δ CBD
AB/CB = BC/BD = AC/CD
AB/12 = 12/9 = AC/6
AB = 16 cm and AC = 8 cm
AD = AB – BD = 16 – 9 = 7 cm
∴ Perimeter of Δ ABC = 7 + 6 + 8 = 21 cm
∴ Perimeter of Δ BDC = 9 + 6 + 12 = 27 cm
∴ Required ratio = 21/27 = 7/9

Hence, option 1.

24.



Δ PQR is an equilateral triangle and PS is the diameter.
∴ $m\angle$PQS = $m\angle$PRS = 90° (angles subtended in a semi-circle)
PS bisects QPS as it is the median of ΔPQR
$m\angle$PMQ = $m\angle$PMR = 90°
∴ $m\angle$QPS = $m\angle$RPS = 30°
∴ $m\angle$PSQ = $m\angle$PSR = 60°

Radius = r
∴ PS = 2r
As Δ PQS, Δ PQM, Δ MQS are 30°-60°-90° triangles,
$\text{QS} = r, \text{PQ} = \sqrt{3r}$
$\text{Similarly, RS} = r, \text{PR} = \sqrt{3r}$
$\therefore$ Perimeter of quadrilateral PQRS $= 2r + 2\sqrt{3} = 2r(1 + \sqrt{3})$
Hence, option 1.

25. n will be of the form 11ab, where a and b are odd numbers.
We are looking for all n's divisible by 3.
∴ 1 + 1 + a + b = 3 or 9 or 12 or 15 or 18
∴ a + b = 1 or 4 or 7 or 10 or 13 or 16
∴ a + b = 1 or 7 or 13 is not possible as the sum of two odd numbers cannot be odd.
∴ (a, b) = (1, 3), (3, 1), (1, 9), (3, 7), (5, 5), (7, 3), (9, 1), (7, 9), (9, 7)
∴ 9 elements of S are divisible by 3.

Hence, option 1.